Radiation Efficiency of Photoreactors

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Theoretical expressions are derived for the fraction of the energy, emitted from a UV lamp, which strikes the reactor wall in systems where a reflecting surface is included. Two geometries used in photoreactors are considered: a cylindrical lamp and tubular reactor located at the foci of an elliptical reflector, and a trough reactor over which is a cylindrical lamp located at the focus of a superimposed parabolic reflector. The predicted efficiencies are low, particularly for the elliptical reflector—reactor system, because of the energy that escapes through the ends of the system. Experimentally determined efficiencies for the elliptical type are in reasonable agreement with predicted values, suggesting that the theoretical method may be used for estimating efficiencies for different reactor systems.

The electrical energy requirement is a major cost in photoreactors because the UV energy adsorbed by the reacting solution is but a small fraction of the energy input to the lamp. This low fraction can be considered to be the product of three efficiencies: the fraction η_1 of the energy input which is emitted in the UV wavelength region of the lamp, the fraction η_2 of the emitted UV energy which reaches the surface of the reacting solution, and the fraction η_3 of the incident energy which is absorbed by the solution. The first efficiency is a characteristic of the lamp; it is inversely dependent upon the fraction of energy input converted to long wavelengths (heat). The third efficiency depends upon the absorption characteristics of the solution, the absorptivity, concentrations of absorbing species, and the radiation path length. This paper is concerned with the second efficiency, which depends upon the geometry of the lamp-reflector-reactor system. As such it is under control of the designer, and proper consideration of this efficiency can lead to reduced costs for photoreactors.

It has been customary to consider that radiation from a tubular lamp is solely in the direction perpendicular to the axis of the lamp. This procedure is not satisfactory when η_2 is to be evaluated, since the lamp emits radiation in all directions. Significant amounts of radiation do not reach the reactor, and η_2 is much less than unity, when rays leave the lamp at an angle such that they do not strike the reactor directly, or after reflection. This was recognized by Dranoff (*1, 2*) who derived an equation for the intensity at any point in an annular reactor (containing a lamp at the center line and with no reflector) by considering rays from the lamp reaching the point from all directions. Our objective is to illustrate a procedure for calculating the efficiency η_2 by considering two reactors: a trough type with parabolic reflector, which has advantages for some large scale applications, and an elliptical reflector with lamp and tubular reactor at the foci, which is a type used for laboratory scale, flow photoreactors and for which experimental η_2 values are also available.

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TROUGH REACTOR

A trough reactor with a lamp located at the focus of a superimposed, parabolic reflector is shown in Figure 1. The reacting solutions flows in the z direction through the rectangular cross section of the trough. The problem is to find the local intensity at any point on the surface of the reacting solution, and the average value for η_2 in terms of the dimensions of the system. Energy reaches any point on the surface by direct rays from the lamp and by reflected rays from the parabolic reflector, both arriving from all directions. The main steps in reaching the solution, by analytic geometry, are summarized for the trough reactor,

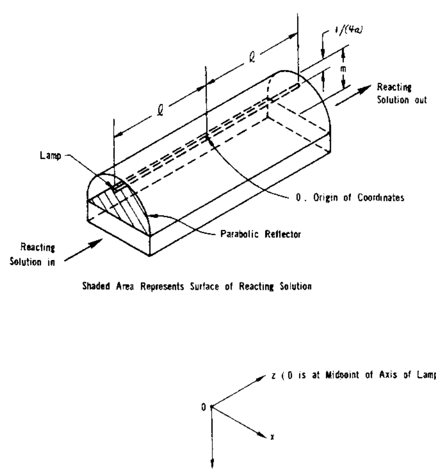

Fig. 1. Trough reactor with parabolic reflector.

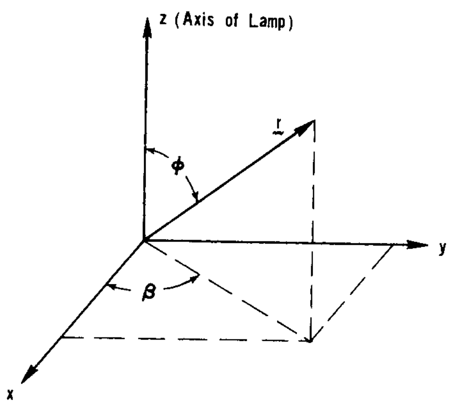

Fig. 2. Coordinate system.

while only final results are given for the elliptical system. Details of the derivations for both cases are available (3).

Reflection from a Parabolic Cylinder

The system shown in Figure 1 can be defined in terms of m, l, and a, where a is defined by the following equation for the parabolic reflector:

$$y - a\,x^2 + \frac{1}{4a} = 0 \quad \text{for all } z \tag{1}$$

The determination of the direction of the reflected ray is simplified by transforming Equation (1) to spherical coordinates, shown in relation to x, y, z in Figure 2. Then the unit vector δ, representing the direction of a ray from the lamp inclined at an angle ϕ with the axis of the lamp, is given by

$$\boldsymbol{\delta} = \frac{\mathbf{r}}{|\mathbf{r}|} = (\sin\phi\cos\beta)i + (\sin\phi\sin\beta)j + (\cos\phi)k \tag{2}$$

The general expression for the normal to the reflector can be written (4), as the unit normal vector, as follows:

$$\mathbf{n} = \left[\frac{1}{2(1+\sin\beta)}\right]^{1/2}[(1+\sin\beta)i - (\cos\beta)j] \tag{3}$$

The unit vector $\boldsymbol{\delta}'$ for the reflected ray is given from the law of reflection (5) in terms of the unit vector of the incident ray, and the unit vector of the normal, as follows:

$$\mathbf{n}\times\boldsymbol{\delta} = \mathbf{n}\times\boldsymbol{\delta}' \tag{4}$$

Solving Equation (4) for the nontrivial direction cosines of $\boldsymbol{\delta}'$, we get

$$\boldsymbol{\delta}' = (\sin\phi)j + (\cos\phi)k \tag{5}$$

Equation (5) shows that the reflected ray lies in the $y\,z$ plane and has a direction independent of β. Also it can be shown (3) that all rays leaving the lamp with an inclination ϕ are reflected in such a way that they are coplanar. This is illustrated in Figure 3, where O represents a point on the axis of the lamp.

Energy Transfer by Reflection

Figure 1 shows the geometry of the trough reactor: length $2l$, with lamp located at $x = y = 0$ in the z axis. The irradiated surface of the reacting solution is the rectangle in the xz plane at a distance $y = m - 1/(4a)$, where a is the proportionality constant in Equation (1). The length of the irradiated rectangle is $2l$, and its width, from Equation (1), at $y = m - 1/(4a)$ is $2(m/a)^{1/2}$.

The distribution of intensity of reflected radiation reaching the surface will not be uniform in either the x or z directions. However, $I(x, z_1)$ can be found by evaluating the intensity distribution along the x axis, normalizing it, and multiplying the result by the distribution along the z axis. This is possible because it has been shown that all rays from one point on the lamp, with an inclination ϕ, are reflected into one plane and that these rays reach the reacting surface in a line parallel to the x axis.

To find the x distribution, we suppose radial light rays from the lamp at any z. Then if the intensity at unit distance from the lamp axis is I, the intensity at any distance r will be

$$I(r) = \frac{I}{|\mathbf{r}|} \tag{5a}$$

The intensity at the reflector is

$$I_f(r) = \frac{I}{|\mathbf{r}|}(\mathbf{n}\cdot\boldsymbol{\delta}) \tag{6}$$

After reflection to the reacting surface, the intensity will be

$$I_s(x) = \frac{-I_f(r)}{(\mathbf{n}\cdot\boldsymbol{\delta}')} \tag{7}$$

Utilizing Equations (2), (3), and (6) in (7), we get

$$I_s(x) = \frac{I}{a\,x^2 + 1/(4a)} \tag{8}$$

If this expression for $I_s(x)$ is divided by the integral of $I_s(x)$ from $x = -(m/a)^{1/2}$ to $(m/a)^{1/2}$, the normalized x distribution of intensity from the reflected rays is obtained:

$$I_s^n(x) = \left[\frac{1}{4\tan^{-1}(2\sqrt{am})}\right]\left(\frac{1}{ax^2 + 1/4a}\right) \tag{9}$$

The consequences of Equation (5), as stated previously, mean that the z distribution of intensity can be evaluated by transforming the actual system into an equivalent, two-dimensional model. As depicted in Figure 3, the rays leaving the lamp (at origin O) at an angle ϕ, and which strike the reflector at R and are reflected to the surface of the reacting solution, can be imagined to originate at O'. The distance $O'R$ is equal to $|\mathbf{r}|$. Hence, the intensity $I_s(z_1)$ at the surface can be formulated by using the two-dimensional transformation of Figure 4, where dz and dz_1 represent differential lengths along the lamp and along the surface of the solution, respectively. The distance $d = m + 1/(4a)$ for the reflected rays.

By using the view-factor approach (6), the rate of energy emission in length dz through angle $d\phi$ in a direction ϕ is

$$dQ = \frac{q}{2}(\sin\phi)\,d\phi dz \tag{10}$$

where q is the total rate of energy emission per unit length of lamp. The fraction of the emitted energy intercepted by length dz_1 is $(dz_1)\sin\phi/(p\,d\phi)$. This quantity is the projection of dz_1 perpendicular to y divided by the arc length formed by the angle $d\phi$ with a circle of radius p whose center is at dz (Figure 4). Multiplication of this fraction by $d\phi$ gives the energy emitted from dz and intercepted by dz_1:

$$dQ_1 = \frac{q}{2}\,\frac{\sin^2\phi}{p}\,dz\,dz_1 \tag{11}$$

Since the x distribution of intensity has been normalized, in calculating the z distribution of energy one must consider that only a fraction of the emission from the lamp will strike the parabolic reflector. From Figure 3 this fraction will be

$$\kappa = \frac{\pi + 2\tan^{-1}\left[\dfrac{m - 1/4a}{(m/a)^{1/2}}\right]}{2\pi} \tag{12}$$

Then the intensity at point z_1, due to radiation from dz, is given by the derivative of κQ_1:

$$d\,I_s(z_1) = \frac{d(\kappa Q_1)}{dz_1} = \frac{\kappa q}{2}\,\frac{\sin^2\phi}{p}\,dz \tag{13}$$

The total intensity at z_1 is obtained by integrating Equation (13) from $z = -l$ to $+l$. By using the expressions for $\sin\phi$ and p indicated by the geometry of Figure 4, the integrated result is

$$I_s(z_1) = \frac{\kappa q}{2}\left\{\frac{l - z_1}{[(l - z_1)^2 + (m + 1/4a)^2]^{1/2}} + \frac{l + z_1}{[(l + z_1)^2 + (m + 1/4a)^2]^{1/2}}\right\} \tag{14}$$

Finally, the distribution of intensity at the surface of the reacting solution, for reflected light, is given by the product of Equations (9) and (14). If Equation (12) is employed for κ, the result is

$$I(x, z_1) = \frac{\tau}{ax^2 + 1/4a}\left\{\frac{l - z_1}{[(l - z_1)^2 + (m + 1/4a)^2]^{1/2}} + \frac{l + z_1}{[(l + z_1)^2 + (m + 1/4a)^2]^{1/2}}\right\} \tag{15}$$

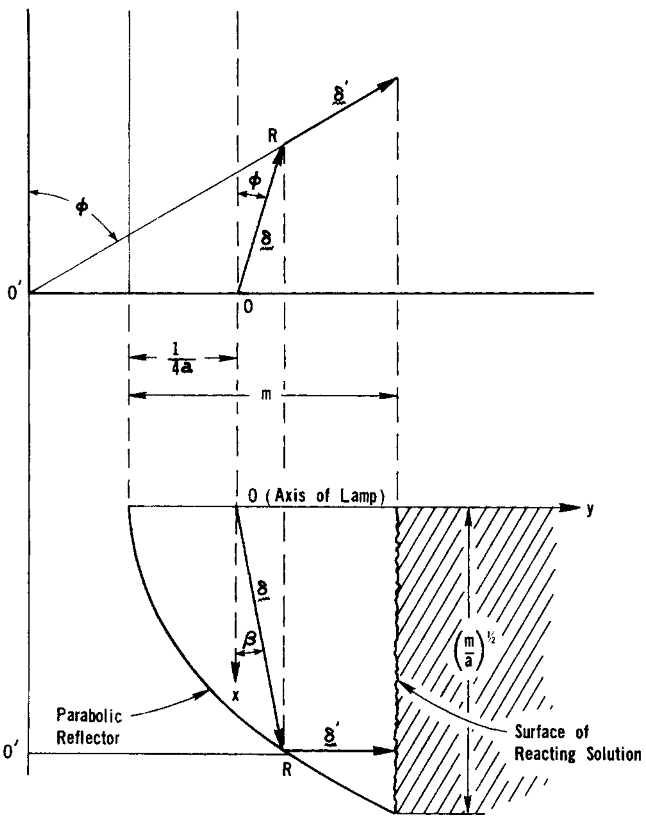

Fig. 3. Emitted and reflected rays in actual reactor system and in transformed two-dimensional model.

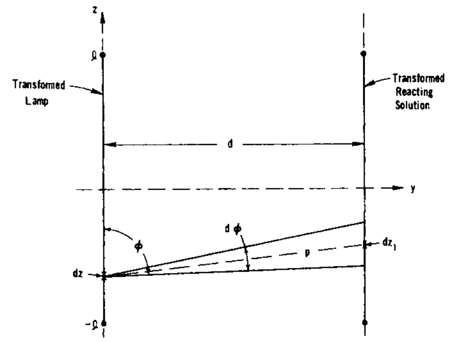

Fig. 4. Transformed geometry of trough reactor system.

where

$$\tau = \frac{q\left[\dfrac{1}{2} + \dfrac{1}{\pi}\tan^{-1}\left(\dfrac{m - 1/4a}{(m/a)^{1/2}}\right)\right]}{8\tan^{-1}(2\sqrt{am})} \tag{16}$$

Direct Energy Transfer

The direct transfer of energy from lamp to reacting solution also may be formulated as the product of a normalized x distribution and a z distribution. The x distribution is

$$I_{\mathrm{dir}}(x) = \frac{I}{|\mathbf{r}|}\cos\left(\frac{\pi}{2} - \beta\right) = \frac{I}{|\mathbf{r}|}\sin\beta \tag{17}$$

where $|\mathbf{r}|$ is now the distance between the lamp and a point, on the surface of the reacting solution, whose coordinates are x_1, $m - 1/4a$, O). Writing Equation (17) in terms of x and m, and normalizing the result, with respect to the integrated value from $x = -(m/a)^{1/2}$ to $(m/a)^{1/2}$, we get

$$I^n_{\mathrm{dir}}(x) = \frac{1}{2\tan^{-1}\left[\dfrac{(m/a)^{1/2}}{m - 1/4a}\right]}\;\frac{m - 1/4a}{x^2 + (m - 1/4a)^2} \tag{18}$$

For the z distribution, the analogue of Equation (14) is valid, where $(1 - \kappa)$ is substituted for κ and $m - 1/4a$ replaces $1 + 1/4a$. Multiplication of this result by Equation (18) gives the intensity of direct radiation from the lamp to z_1:

$$I_{\mathrm{dir}}(x, z_1) = \frac{\tau_{\mathrm{dir}}\,(m - 1/4a)}{x^2 + (m - 1/4a)^2}\left\{\frac{l - z_1}{\left[(l - z_1)^2 + \left(m - \dfrac{1}{4a}\right)^2\right]^{1/2}} + \frac{l + z_1}{\left[(l + z_1)^2 + \left(m - \dfrac{1}{4a}\right)^2\right]^{1/2}}\right\}. \tag{19}$$

where

$$\tau_{\mathrm{dir}} = \frac{q\left\{\dfrac{1}{2} - \dfrac{1}{\pi}\tan^{-1}\left[\dfrac{m - \dfrac{1}{4a}}{(m/a)^{1/2}}\right]\right\}}{4\tan^{-1}\left\{\dfrac{(m/a)^{1/2}}{m - \dfrac{1}{4a}}\right\}} \tag{20}$$

Efficiency η_2

If the reflectance of the reflector is ρ, the total intensity at any point x, z_1 on the surface of the reacting solution will be

$$I_t(x, z_1) = \rho\, I(x, z_1) + I_{dir}(x, z_1) \tag{21}$$

with $I(x, z_1)$ and $I_{dir}(x, z_1)$ given by Equations (15) and (19). Then the fraction of the radiation from the lamp which reaches the reaction surface is

$$\eta_2 = \frac{\int_{-(m/a)^{1/2}}^{(m/a)^{1/2}} \int_{-l}^{l} I_t(x, z_1)\, dx\, dz_1}{\int_{-l}^{l} \int_0^{\pi} \frac{q}{2} (\sin \phi)\, d\phi\, dz} \tag{22}$$

The denominator is the integral of Equation (10) and has a value equal to F_t, the total UV energy emitted by the lamp. Carrying out the integrations, we get

$$\eta_2 = \rho \left\{ \frac{1}{2} + \frac{1}{\pi} \tan^{-1} \left[\frac{m - 1/4a}{(m/a)^{1/2}} \right] \right\} \frac{(w^2 + 1)^{1/2} - 1}{w} \tag{23}$$

$$+ \left\{ \frac{1}{2} - \frac{1}{\pi} \tan^{-1} \left[\frac{m - 1/4a}{(m/a)^{1/2}} \right] \right\} \frac{(w^2_{dir} + 1)^{1/2} - 1}{w_{dir}} \tag{23}$$

where

$$w = \frac{2l}{m + 1/4a} \tag{24}$$

$$w_{dir} = \frac{2l}{m - 1/4a} \tag{25}$$

Equation (23) gives η_2 in terms of the dimensions m, l, and a of the trough reactor-reflector system, as shown in Figure 1, and in terms of the reflectance ρ. As expected, η_2 increases as l increases and m decreases. For reflectances of 0.8 or more, Equation (23) predicts efficiencies of the order of 70% when the reactor length $(2l)$ to height (m) ratio is about 20. When this ratio is but 2 to 3, η_2 drops to 30 to 40%.

ELLIPTICAL REACTOR

This type of reactor with its elliptical reflector, tubular lamp, and tubular reactor located at the foci, is shown in Figure 5. The radiating length of the lamp $(2l_l)$, irradiated length of reactor $(2l_r)$, and the reflector are located so that their midlengths are in the same horizontal plane. In this system η_2 can be low because, for small l_l and large α, much radiation from the lamp is lost through the nonreflecting elliptical ends of the reflector.

The analytical treatment can be carried out in the same way as described for the trough reactor. The results are

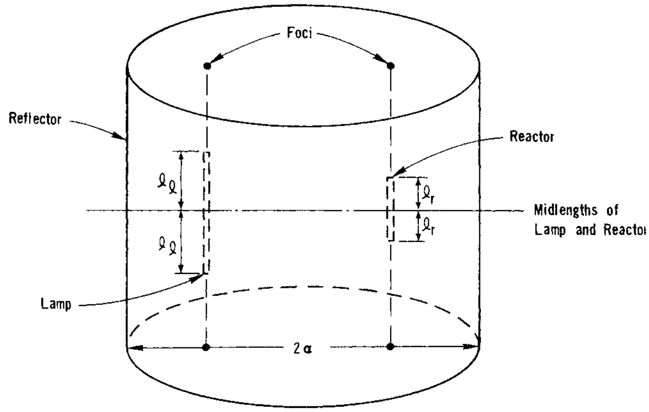

Fig. 5. Location of lamp and reactor in elliptical reflector.

simple when $l_l = l_r = l$. In this case the intensity distribution along the reactor (line z_1) from reflected rays is given by

$$I(z_1) = \frac{q}{2} \left\{ \frac{l - z_1}{[(l - z_1)^2 + (2\alpha)^2]^{1/2}} + \frac{l + z_1}{[(l + z_1)^2 + (2\alpha)^2]^{1/2}} \right\} \tag{26}$$

For practical values of α and reactor-tube diameter, the contribution of direct radiation would be small. Hence, Equation (26) closely approximates the total intensity. The corresponding expression for the efficiency is

$$\eta_2 = \frac{\left[\frac{l^2}{\alpha^2} + 1 \right]^{1/2} - 1}{l/\alpha} \tag{27}$$

Figure 6 is a plot of η_2 versus l which shows how the efficiency decreases sharply when $l/\alpha < 2$.

When $l_l \neq l_r$, the efficiency is given by

$$\eta_2 = \frac{\left[\left(\frac{l_l + l_r}{2\alpha} \right)^2 + 1 \right]^{1/2} - \left[\left(\frac{l_l - l_r}{2\alpha} \right)^2 + 1 \right]^{1/2}}{l_l/\alpha} \tag{28}$$

For three elliptical-reactor systems used in our laboratory (7 to 9), actinometric data were available for establishing experimental values of η_2 from the following equation:

$$(\eta_2)_{exp} = \frac{I_{tot}\,(2\pi\, R_r)\; 2l_r}{F_t} \tag{29}$$

I_{tot} is the total intensity [Einsteins/(second)(square centimeters)] determined from the actinometer data, and F_t is the rate of UV-energy output of the lamp, as given by the manufacturer, for the wavelength range where there is absorption by the actinometer solution (uranyl sulfate —

Table 1. Radiation Efficiency of Elliptical Reactor Systems

Lamp		Reactor		Reflector		Efficiency, η_2	
Power, w.	Length, $2l_l$, cm.	Length $2l_r$, cm.	Diameter $2R_r$, cm.	Major axis 2α, cm.	Wavelength range, A	Exp. Equation (29)	Calc. Equation (28)
Uviarc*	15.2	10.0	0.2	87	2,250 to 4,900	0.0054	0.057
Uviarc*	15.2	1.0	1.0	87	2,250 to 4,900	0.0017	0.0058
189A-10†	30.0	22.5	2.0	61	2,224 to 4,358	0.094	0.177

* General Electric Co. U3, 360 w. lamp.
† Hanovia 1,200 w. lamp.

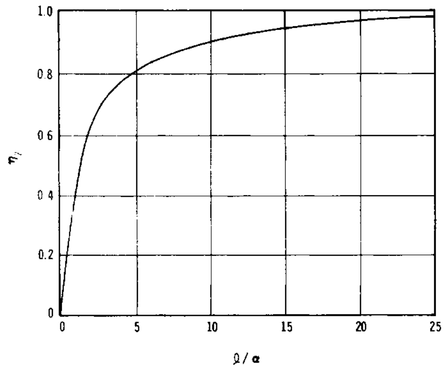

Fig. 6. Efficiency of radiation transfer in elliptical reactor system ($l_l = l_r = l$).

oxalic acid). Experimental η_2, and values calculated from Equation (28), are shown in Table 1. Also included are the calculated efficiencies and the dimensions of the systems.

In all cases the efficiencies are very low because l_l/α is small. In this region slight errors in geometry cause large changes in efficiency (see Figure 6). The calculated results are higher than the experimental values, probably due to the several assumptions used in the theory. These simplifications include the following: exact elliptical shape for the reflector, exact location of reactor and lamp at the foci, constant energy output along length of lamp, and line source and sink for the tubular lamp and reactor. Nevertheless, Equation (28) does predict the low level of efficiencies that are observed. The ratio of η_2 values becomes nearer to unity as the reactor diameter increases, probably because errors in alignment at the foci become less important. In addition to the assumptions mentioned, the calculated values of η_2 are high because the reflectance has been taken as unity in Equation (28) while it is probably about 0.7 to 0.8. Also the experimental η_2 depends upon lamp manufacturer data for F_t, which may be approximate. Finally, the contribution of direct radiation from lamp to reactor has been neglected in Equation (28). For the reactor tube of 2 cm. diameter, direct radiation would increase the efficiency a small ($\sim 1\%$) but finite amount.

In summary, the proposed method should be useful for estimating the efficiency of energy transfer for various types of photoreactors, when experimental, actinometric data are unavailable.

ACKNOWLEDGMENT

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NOTATION

a = characteristic constant of parabola, Equation (1), cm.$^{-1}$
d = distance, cm.
dQ = energy rate emitted from lamp length dz in an angle $d\phi$, w./sec.
F_t = UV-energy output of lamp in range of wavelength for which actinometer absorbs radiation, w.
I = energy rate per unit length at unit distance; $I(r)$ is the energy at distance r, w./cm.
$I(z_1)$ = energy rate received per unit length of reactor at a distance z, w./cm.
$I(x)$ = energy rate per unit length at a distance x from center line ($x = 0$) of trough, w./cm.
$I(x, z_1)$ = intensity of energy at surface of reacting solution, w./sq.cm.
I_{tot} = total intensity reaching surface of reacting solution, w./sq.cm.
l = half-length, cm.
m = distance from top of parabolic reflector to reacting solution, cm.
x, y, z = rectangular coordinates
$\mathbf{n}$ = unit normal vector at reflector (at point R)
O = origin of coordinates (at midlength of axis of lamp)
p = path length of ray, cm.
q = energy rate per unit length of lamp, w./(cm.)(sec.)
r = distance from origin of spherical coordinates, cm.
R = point of reflection (Figure 3)
R_r = radius of reactor tube, cm.
w = defined by Equations (24) and (25)

Greek Letters

2α = length of major axis of ellipse, cm.
β = angle between x axis and projection of $\mathbf{r}$ in xy plane (Figure 2)
$\mathfrak{S}$ = unit vector of incident ray to the reflector; $\mathfrak{S}'$ unit vector for the reflected ray
η_2 = fraction of energy leaving lamp which strikes surface of reaction solution
ϕ = angle between z axis and position vector $\mathbf{r}$ (Figure 2)
κ = fraction of energy reflected
ρ = reflectance
τ = function of q for reflected rays, defined by Equation (16)
τ_{dir} = function of q for direct rays, defined by Equation (20)

Subscripts

1 = reactor
f = reflector
s = surface of reacting solution
dir = direct radiation from lamp to surface of reacting solution
t = total of direct plus reflected radiation

Superscript

n = normalized quantity

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